Relating Period and Cohort Fertility

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APPENDIX

A. Derivation of Eq(7) [Linear fertility trajectory]

$$A_c(T) = \int x \, TFR(T+x) \, c(x) \, dx \, / \, CFR(T)$$

$$= \int x \, [R + a(T+x)] \, c(x) \, dx \, / \, CFR(T) = \int \{x \, [R + aT]c(x) + ax^2 \, c(x)\} \, dx \, / \, CFR(T)$$

$$= \{(R+aT) \, [\int x \, c(x) \, dx] + a \int x^2 \, c(x) \, dx\} \, / \, CFR(T) \qquad (A.1)$$

with all integrals ranging over ages 15 to 45. The final equality expresses the cohort mean age of fertility as a function of the first and second moments of proportional fertility schedule c(x). The first moment is the mean, denoted μ, equals $\int x \, c(x) \, dx$. The second moment about the mean is the variance (Var). Specifically,

$$Var = \int (x-\mu)^2 \, c(x) \, dx$$

$$= \int x^2 \, c(x) \, dx - 2\mu \int x \, c(x) \, dx + \mu^2$$

$$= \int x^2 \, c(x) \, dx - \mu^2 \qquad (A.2)$$

Rewriting Eq(A.2),

$$\int x^2 \, c(x) \, dx = Var + \mu^2 \qquad (A.3)$$

Substituting Eq(A.3) into Eq(A.1) using Eq(5) yields

$$A_c(t) = [(R+aT)\mu + a(Var+\mu^2)] \, / \, CFR(T) = \{[R+a(T+\mu)]\mu + aVar] \, / \, TFR(T+\mu)$$

$$= \mu + aVar/CFT(T) \qquad (A.4)$$

B. Derivation of Eqs(12)-(13) [Quadratic fertility trajectory]

The derivation essentially follows from integrating the quadratic TFR function over the reproductive years, ages 15 to 45. Thus, using Eq(A.3)

$$CFR(T) = \int [R + a(T+x) + b(T+x)^2] \, c(x) \, dx = (R+aT+bT^2) + \mu(a+2bT) + b \int x^2 \, c(x)dx$$

$$= TFR(T) + 2bT\mu + a\mu + b\mu^2 + b \, Var$$

$$= TFR(T+\mu) + b \, Var \qquad (A.5)$$

For the quadratic mean age, the third moment about the mean, i.e. the skew, is needed. Here, we assume that the fertility function is symmetric (e.g. a normal curve or the parabola of Eq(6)), and write the definition of the skew, with its value set to zero, as

$$0 = \int (x-\mu)^3 c(x)dx = \int x^3 c(x)dx - \int 3\mu x^2 c(x)dx + \int 3\mu^2 x\, c(x)dx - \int \mu^3 c(x)dx \qquad (A.6)$$

Rewriting (A.6) to find the third moment,

$$\int x^3 c(x)\, dx = 3\mu[Var+\mu^2] - 3\mu^3 + \mu^3 = 3\mu Var + \mu^3 \qquad (A.7)$$

Integrating for the quadratic cohort mean age of fertility, using Eqs(A.5) and (A.7)

$$A_c(T) = \int x\, [R + a(T+x) + b(T+x)^2]\, c(x)\, dx\, /\, CFR(T) \qquad (A.8)$$

$$= \{\, \mu[R+aT+bT^2] + (a+2bT)[Var+\mu^2] + b[3\mu Var + \mu^3]\, \} / CFR(T)$$

$$= \{\, \mu\, TFR(T) + \mu^2 (a+2bT) + b\mu^3 + b\mu Var + Var[\, a+2bT+3b\mu - b\mu]\, \} / CFR(T)$$

$$= \{\, \mu\, CFR(T) + Var\, (a+2bT+2b\mu)\, \} / CFR(T) = \mu + Var\, [a+2b[T+\mu)] / CFR(T)$$

C. <u>Derivation of Eqs(15) and (16) [Cubic fertility trajectory]</u>

With a cubic fertility trajectory, Eq(14) implies that

$$TFR(T+\mu) = R + a(T+\mu) + b(T^2 +2T\mu+\mu^2) + d(T^3 +3T^2 \mu+3T\mu^2 +\mu^3)$$

$$= TFR(T) + \mu(a+2bT+3dT^2) + \mu^2 (b+3dT) + d\mu^3 \qquad (A.9)$$

Again assuming a zero skew, the cohort fertility rate can be found by integrating Eq(14) using Eqs(A.4), (A.7), and (A.9). Thus

$$CFR(T) = \int c(x)\, [\, R + aT + ax + bT^2 +2bTx+bx^2 + dT^3 +3dT^2 x+3dTx^2 +dx^3]\, dx$$

$$= TFR(T) + (a+2bT+3dT^2)\, \mu + [b+3dT]\, (\mu^2 + Var) + d\, (3\mu Var + \mu^3)$$

$$= TFR(T+\mu) + Var\, [\, b + 3d(T+\mu)] \qquad (A.10)$$

For the cubic cohort mean age, we need to know the fourth moment about the mean, i.e. the kurtosis. Assume that c(x) has zero skew and the kurtosis of a normal curve. From Kendall and Stuart (1958, Vol 1, p67-74) and Keyfitz (1977, Chap. 5), we can write

$$\int x^4 c(x)\, dx = 3\, Var^2 + 6\mu^2\, Var + \mu^4 \qquad (A.11)$$

Using Eqs(14), (A.4), (A.7), and (A.9)-(A.11), along with the definition of the CFR in Eq(3),

$$A_c(T) = \int x\, c(x)\, [R+aT+ax+bT^2+2bTx+bx^2+dT^3+3dT^2x+3dTx^2+dx^3]\, dx / CFR(T)$$

$$= \{\mu\, TFR(T) + (a+2bT+3dT^2)[\mu^2 + Var] + (b+3dT)[3\mu Var+\mu^3] + d\,[3Var^2+6\mu^2 Var+\mu^4]\} / CFR(T)$$

$$= \{\mu\, TFR(T+\mu) + Var\,(a+2bT+3dT^2) + 3\mu Var(b+3dT) + d(3Var^2+6\mu^2 Var)\} / CFR(T)$$

$$= \{[\mu\, TFR(T+\mu) + \mu Var(b+3d[\mu+T])] + 2\mu Var(b+3dT) - 3d\mu^2 Var + Var(a+2bT+3dT^2) + 3dVar[2\mu^2 + Var]\} / CFR(T)$$

$$= \{\mu\, CFR(T) + 2\mu Var(b+3dT) + Var(a+2b(T+\mu) +3dVar(T+\mu)^2) + Var^2\} / CFR(T)$$

$$= \mu + Var\,\{a+2b(T+\mu)+3d\,[(T+\mu^2) + Var]\} / CFR(T) \qquad (A.12)$$

D. Derivation of the parabolic curve in Eq(6)

A simple parabola can capture the essential nature of the fertility curve. With the c(x) function 0 at ages 15 and 45, and with an area of 1 between those ages, the curve is specified by the equations

$$0 = w + 15y + (15)^2 z$$

$$0 = w + 45y + (45)^2 z$$

$$1 = \int (w+yx+zx^2)\, dx = wx + yx^2/2 + zx^3/3 \qquad (A.13)$$

where the last equality is evaluated for x at ages 45 and 15. Solving those equations yields the real solution

$$w = -3/20; \quad y = (1/75); \quad \text{and } z = (-1/4500) \qquad (A.14)$$

With total fertility changing linearly, R = 1, and a = 0.02, the t = 30 period and T = 0 cohort age curves of fertility are shown in Figure 1. Integrating the fertility function over 5-year intervals yields the 5-year values plotted, i.e.

Age	Period f(x,5)	Cohort f(x.5)	Δ (Period – Cohort)
15-19	.118519	.101157	.017362
20-24	.296296	.269213	.027083
25-29	.385185	.373380	.011805
30-34	.385185	.396990	− .011805
35-39	.296296	.323379	− .027083
40-44	.118519	.135881	− .017362

The symmetry about age 30 of the period/cohort differences is apparent.

E. Derivation of Eq(19) for the Average Cohort Fertility

With the constant c(x) of Eq(6) and the cubic trajectory of Eq(14), Eq(18) leads to

$$ACF(t) = \int \{ TFR(t+\mu-x) + Var[b+3d(t+\mu-x)] \} c(x)\,dx$$

$$= R + \int a(t+\mu-x)\, c(x)dx + \int b(t+\mu-x)^2 c(x)dx + \int d(t+\mu-x)^3 c(x)dx + Var[b+3dt] \quad (A.15)$$

where the integrals range over ages 15 to 45. Using Eqs(A.4), (A.7) and (A.9),

$$ACF(t) = R + at + b(t+\mu)^2 - 2b(t+\mu)\mu + b(Var+\mu^2) + d(t+\mu)^3 - 3d(t+\mu)^2\mu + 3d(t+\mu)(Var+\mu^2) - d[3\mu Var+\mu^3) + Var(b+3dt)$$

$$= R + at + bt^2 + dt^3 + Var(2b+6dt) = TFR(t) + 2Var(b+3dt) \quad (A.16)$$

From Eq(15) it follows that

$$ACF(t) = CFR(T) + Var(b+3dt) \quad (A.17)$$

F. Sources for United States Fertility Data, 1917 – 2019

For over 45 years, the U.S. National Center for Health Statistics has assembled and published a sizeable amount of data on both period and cohort fertility. The foundational effort was the Heuser (1976) volume *Fertility Tables for Birth Cohorts by Color: United States 1917-1973*, which was the source used for those years. Birth rates by age of mother for the years 1970-2015 are provided by Martin et al (2017), the years 2016-2018 by Martin et al (2019), and the year 2019 from Hamilton et al (2020). Another useful source was Hamilton and Kirmeyer (2017).

Cohort total fertility rates, when not published, were calculated from the age-specific birth rates in the above sources, especially Heuser (1976), and from Hamilton and Cosgrove (2010). The experience of cohorts, up to the birth cohort of 1984, was completed using the experience of the year 2019. Actual data were always used through age 35.

Period mean ages of fertility (A_p) for the years 1917 through 2001 were taken from Schoen (2004), and were calculated, for the years 2002-2015 from Martin et al (1917), for the years 2016-2018 from Martin et al (2019), and for the year 2019 from Hamilton et al (2020). The variance of fertility, for the years 1919-1965 was taken from, or linearly interpolated from, data in Keyfitz and Flieger (1968), and for the years 1966-1985 from Keyfitz and Flieger (1990). Variances for years after 1985 were calculated from the published age-specific birth rates for those years.

G. Calculations to Estimate Cohort Fertility

Five different CFR estimates were calculated, using alternative model specifications. The first alternative simply compared the observed CFR(T) to the corresponding TFR, and is shown in Figure 3. That assumption of linearity in the TFR is rather crude, and is off by an average of 0.196 children. Using a base CFR value of 2, that is an average error of 9.8%.

The next 2 estimates assume that fertility in an interval around focal year t follows a cubic curve, and uses the adjustment to TFR shown in Table 1. To fit a cubic curve, the length of the interval of estimation needs to be specified. Less than 20 years seems too short, while more than 30 years would be very data demanding. Intervals of both 20 and 30 years were considered. First, at each time point, the 20-year cubic estimate rescaled time t to time zero and fit the cubic curve to TFRs for times −10, −5, +5, and +10. The estimated CFR(T) is then just [TFR(t) + b_{20} Var], where b_{20} is the coefficient of the quadratic term in the cubic equation derived from the TFR values of the 20-year fitting interval. Solving for the cubic coefficients R, a, b, and d, using the known TFR values and four equations of the form of Eq(14), yields

$$b_{20} = [TFR(-10) - TFR(-5) - TFR(5) + TFR(10)] / 150 \quad \text{(A.18))}$$

The 30-year estimate fits the cubic curve to TFRs for times −15, −5, +5, and +15. That 30 year interval matches the length of the reproductive age span, and gives the cubic a broader base. The solution for b_{30} is

$$b_{30} = [TFR(-15) - TFR(-5) - TFR(5) + TFR(15)] / 400 \quad \text{(A.19)}$$

(See **Appendix Supplement 3**.) Quadratic parameters b_{20} and b_{30} are both estimated for the cubic curve appropriate to each calendar year. Alternative calculations were made with the fitted TFR values calculated as the arithmetic means of the TFRs for the 5 years centered on the fitted points. Those results were very similar.

The last 2 estimates use those b_{20} and b_{30} values but with graduated period TFR values. Weighted moving averages are a classic technique for smoothing a time series to eliminate data perturbations. Even vital statistics data, which are based on a large number of cases, exhibit irregularities. The standard graduation approach minimizes deviations from a cubic curve and uses 2m+1 weights applied to the focal year and to values m years before and after that focal year. The value of m is generally set at between 3 and 11. Here, with our long time series, m was set at 10, and the 21-term weighted moving average in Hoem and Linnemann (1988, Table A 2.8) was used to smooth the TFR series and produce the gTFR series. **Appendix Table 1** and **Appendix Supplement 5** provide yearly values for b_{20}, b_{30}, Var, the graduated TFR series, and the errors of estimate.

Appendix Table 2 shows that estimates from either b_{20} or b_{30} work about equally well. The graduated TFR values produce estimates averaging about 0.01 better than the ungraduated TFR values, and substantially reduce the number of large errors. On a base of 2, the graduated TFR series yields estimation errors averaging 7.2%, while the ungraduated TFRs yield errors averaging 7.8% (i.e. 8% larger). Appendix Table 1 shows that estimates for years after 2000 are particularly prone to error, with many differences of 0.20 to 0.24.

The 4 cubic estimated CFR curves are quite similar. Because the ungraduated b_{30} curve is both easier to calculate and more demographically rooted, it is shown in Figure 3.

H. Projecting Cohort Fertility

To find the cubic parameters to adjust TFR(t) to approximate CFR(T) using Eq(15), equations of the form of Eq(A.19) are used. The approach proceeds year by year, from 2005 through 2019. For every year, TFR values for the years 1989, 1999, 2009, and 2019 are used, but the focal year, i.e. the year that is considered year 0, changes. Let z be an integer from 1 through 15. Then, for year (2004 + z), that year is considered year zero and the set of equations to be solved for R, a, b, and d is

$$\begin{aligned}
TFR(1989) = 2.014 &= R + a(-15-z) + b(-15-z)^2 + d(-15-z)^3 \\
TFR(1999) = 2.008 &= R + a(-5-z) + b(-5-z)^2 + d(-5-z)^3 \\
TFR(2009) = 2.002 &= R + a(5-z) + b(5-z)^2 + d(5-z)^3 \\
TFR(2019) = 1.705 &= R + a(15-z) + b(15-z)^2 + d(15-z)^3
\end{aligned} \qquad \text{(A.20)}$$

For example, for the year 2019, z = 15 and the parameter weights are −30, −20, −10, and 0. Thus for the cohort of 1989-90, behavior through 2034-35 is estimated from data through 2019 via a cubic projection.

The results of the calculations are shown in Table 4 of the paper. The Maple calculation program HyperTrans b30Years1989-2019, for the year 2019, is presented in **Appendix Supplement 4**.

The back projections used the same approach, but based the cubic curve on the years 1917, 1927, 1937, and 1947. The results are shown in Table 5 of the paper. Maple calculation program HyperTrans b30Years1917-1947, for the year 1917, is presented in **Appendix Supplement 6**.

I. Estimating Age-Specific Fertility Proportions for the U. S., 1925, 1950, 1975, and 2000.

For consistency with the assumptions underlying Eq(15) and with conventional demographic practice (e.g. in Keyfitz 1977, Chap. 6), age-specific fertility proportions were estimated using a normal curve parameterization. Calculations were made for the years 1925, 1950, 1975, and 2000 as they represent a reasonable sample of the 1917-2019 study interval.

Fertility proportions were used to facilitate comparisons and to focus on the pattern over age. The desired normal curve is then fully specified by the mean and variance (or standard deviation) of the year's fertility (shown in Table 3 and Appendix Table 1). For each year, that mean and standard deviation were transformed into z-scores by the usual relationship

$$z = (X - \mu) / SD \qquad \text{(A.21)}$$

where SD indicates the standard deviation ($Var^{1/2}$) and X the age group boundary. Here, the age groupings were under 20, 20-24, 25-29, 30-34, 35-39, and 40+. Using a table of areas under the normal curve (see Bohrnstedt and Knoke 1982, Appx C), differences between areas set off by the z-scores were used to determine the age-specific fertility proportions. Graphs of the observed and estimated age-specific fertility rates for the 4 years are shown in Figure 4.

J. Appendix Supplements: Maple programs and Excel worksheet of calculations

Supplement 1. Maple program HyperTrans PCasbrFig1. Calculations for Figure 1.
Supplement 2. Maple program HyperTrans CubicParams. Calculations for Table 2/Figure 2.
Supplement 3. Maple program HyperTrans cubicb30. Calculations for b_{20} and b_{30}.
Supplement 4. Maple program HyperTrans b30Years1989-2019. Calculations for Table 4.
Supplement 5. Excel Worksheet HyperTrans US1917-2019 P/C TFRs #2. Calculations for Appendix Tables 1 and 2 and Table 3; values for Figure 3.
Supplement 6. Maple program HyperTrans b30Years1917-1947. Calculations for Table 5.

Appendix References

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Appendix Table 1. Fertility Measures for the United States, 1927-2009, and Differences Between Observed and Estimated Cohort Total Fertility Rates

					Difference between observed CFR and			
Year	Var	b_{20}	b_{30}	gTFR	TFR+b_{20}	TFR+b_{30}	gTFR+b_{20}	gTFR+b_{30}
1927	46.10	.00052		2.788	−.213		−.177	
1928	45.70	.00174		2.691	−.157		−.187	
1929	45.30	−.00076		2.592	.028		−.032	
1930	45.30	.00195		2.494	−.144		−.105	
1931	45.30	.00408		2.407	−.144		−.150	
1932	44.89	.00444	.00379	2.327	−.113	−.084	−.122	−.093
1933	44.49	.00573	.00364	2.257	−.069	.024	−.153	−.060
1934	44.08	.00608	.00350	2.209	−.182	−.068	−.159	−.046
1935	44.08	.00448	.00382	2.187	−.092	−.063	−.090	−.062
1936	44.08	.00684	.00448	2.177	−.175	−.072	−.207	−.103
1937	42.90	.00755	.00381	2.175	−.224	−.063	−.226	−.065
1938	41.72	.00582	.00410	2.193	−.220	−.148	−.191	−.119
1939	40.54	.00561	.00464	2.230	−.095	−.056	−.154	−.114
1940	40.54	.00633	.00475	2.286	−.143	−.079	−.200	−.136
1941	40.54	.00398	.00376	2.357	−.105	−.096	−.131	−.122
1942	40.09	.00167	.00288	2.436	−.188	−.236	−.069	−.117
1943	39.64	.00182	.00260	2.521	−.245	−.276	−.126	−.157
1944	39.19	.00324	.00241	2.608	−.132	−.099	−.246	−.213
1945	39.19	.00287	.00220	2.701	−.022	.004	−.301	−.275
1946	39.19	.00146	.00109	2.793	−.321	−.307	−.256	−.241
1947	38.57	.00010	−.00025	2.882	−.483	−.469	−.183	−.170
1948	37.95	−.00093	−.00130	2.972	−.226	−.212	−.172	−.158
1949	37.34	−.00097	−.00138	3.061	−.206	−.191	−.231	−.215
1950	37.34	−.00057	−.00212	3.147	−.160	−.102	−.279	−.220
1951	37.34	−.00378	−.00412	3.231	−.119	−.106	−.150	−.137
1952	36.82	−.00590	−.00541	3.315	−.166	−.184	−.194	−.212
1953	36.55	−.00478	−.00501	3.397	−.204	−.196	−.252	−.244
1954	36.03	−.00673	−.00520	3.469	−.213	−.268	−.221	−.276
1955	36.03	−.00887	−.00493	3.528	−.138	−.280	−.167	−.309
1956	36.03	−.00823	−.00547	3.575	−.225	−.324	−.195	−.294
1957	35.46	−.00669	−.00540	3.614	−.321	−.367	−.252	−.298
1958	35.18	−.00793	−.00536	3.631	−.194	−.284	−.195	−.286
1959	34.62	−.00782	−.00576	3.610	−.192	−.264	−.164	−.235

Appendix Table 1. Fertility Measures for the United States, 1927-2009, and Differences Between Observed and Estimated Cohort Total Fertility Rates (con't)

					Difference between observed CFR and			
Year	Var	b_{20}	b_{30}	gTFR	TFR+b_{20}	TFR+b_{30}	gTFR+b_{20}	gTFR+b_{30}
1960	34.62	−.00613	−.00554	3.553	−.209	−.230	.157	−.178
1961	34.62	−.00554	−.00420	3.468	−.191	−.237	−.095	−.141
1962	34.26	−.00619	−.00309	3.364	−.057	−.163	.003	−.103
1963	34.23	−.00565	−.00318	3.244	−.001	−.085	.053	−.031
1964	34.39	−.00510	−.00304	3.108	.040	−.031	.103	.032
1965	35.12	−.00530	−.00292	2.964	.267	.183	.184	.101
1966	35.02	−.00311	−.00200	2.813	.323	.284	.180	.141
1967	34.90	.00037	−.00076	2.661	.207	.246	.071	.110
1968	34.80	.00152	.00003	2.514	.151	.205	.069	.123
1969	34.70	.00294	.00066	2.372	−.005	.074	.046	.125
1970	34.60	.00546	.00179	2.242	−.199	−.072	−.009	.118
1971	34.39	.00645	.00258	2.129	−.135	−.002	−.019	.114
1972	34.18	.00624	.00310	2.033	.057	.164	.017	.125
1973	33.97	.00604	.00343	1.954	.128	.216	.036	.125
1974	33.76	.00498	.00355	1.892	.135	.182	.078	.126
1975	33.55	.00303	.00354	1.843	.239	.222	.141	.124
1976	33.40	.00300	.00392	1.803	.210	.179	.145	.114
1977	33.24	.00384	.00412	1.775	.100	.091	.115	.105
1978	33.09	.00469	.00414	1.759	.078	.097	.080	.098
1979	32.93	.00530	.00383	1.754	−.003	.046	.051	.099
1980	32.78	.00616	.00318	1.762	−.064	.034	.014	.112
1981	33.10	.00488	.00268	1.779	.008	.081	.042	.115
1982	33.42	.00252	.00209	1.798	.074	.089	.103	.118
1983	33.74	.00125	.00184	1.819	.147	.127	.127	.107
1984	34.06	.00010	.00152	1.842	.183	.134	.147	.099
1985	34.38	−.00132	.00142	1.868	.205	.111	.181	.087
1986	34.54	−.00107	.00100	1.894	.209	.137	.152	.081
1987	34.70	−.00075	.00035	1.920	.174	.135	.125	.087
1988	34.87	−.00040	.00023	1.946	.107	.085	.095	.073
1989	35.03	.00005	.00020	1.969	.020	.015	.065	.060
1990	35.19	.00049	−.00005	1.988	−.051	−.031	.042	.061
1991	35.61	.00019	.00008	2.004	−.009	−.005	.050	.054
1992	36.03	.00003	.00017	2.014	.031	.026	.062	.058

Appendix Table 1. Fertility Measures for the United States, 1927-2009, and Differences Between Observed and Estimated Cohort Total Fertility Rates (con't)

					Difference between observed CFR and			
Year	Var	b_{20}	b_{30}	gTFR	TFR+b_{20}	TFR+b_{30}	gTFR+b_{20}	gTFR+b_{30}
1993	36.46	−.00058	−.00025	2.020	.102	.090	.101	.090
1994	36.88	−.00109	−.00053	2.020	.150	.129	.131	.110
1995	37.30	−.00157	−.00092	2.017	.206	.181	.167	.142
1996	37.28	−.00098	−.00097	2.012	.187	.187	.151	.151
1997	37.27	−.00050	−.00090	2.006	.179	.194	.144	.159
1998	37.23	−.00041	−.00103	2.004	.184	.207	.179	.202
1999	37.21	−.00025	−.00096	2.008	.208	.235	.208	.234
2000	37.18	−.00015	−.00087	2.020	.166	.192	.202	.228
2001	37.16	−.00085	−.00107	2.035	.230	.238	.225	.233
2002	37.14	−.00110	−.00113	2.050	.253	.243	.224	.225
2003	37.13	−.00129	−.00102	2.061	.218	.207	.204	.194
2004	37.40	−.00097	−.00073	2.066	.225	.216	.210	.201
2005	37.50	−.00110		2.065	.224		.216	
2006	37.68	−.00086		2.056	.167		.218	
2007	37.75	−.00110		2.043	.142		.220	
2008	38.06	−.00118		2.024	.156		.204	
2009	38.14	−.00134		2.001	.203		.204	

SOURCES: See text, Appendix Sections F and G, and Appendix Supplement 5. Fertility variances (Var) for years through 1985 taken or calculated from Keyfitz and Flieger (1968, 1990) or published rates.

NOTES: See text. Symbols b_{20} and b_{30} are coefficients of the quadratic term in cubic curves fit to TFR values spanning 20 and 30 years, respectively. The symbol gTFR indicates a Total Fertility Rate smoothed by a 21-term weighted moving average.

Appendix Table 2. Assessing the Accuracy of Alternative Estimates of United States Total Fertility Assuming Constant Age-Specific Fertility Proportions and Cubically Varying Period Fertility Levels, 1917-2019

Method	Years Estimated	Average Error	Number of Errors > 0.2	Number of Errors > 0.3	Number of Errors > 0.4
Observed – TFR(t)	1917-2013 (97)	.196	39	17	8
Observed – (TFR(t)+b_{20}Var)	1927-2009 (83)	.157	29	4	1
Observed – (TFR(t)+b_{30}Var)	1932-2004 (73)	.154	23	4	1
Observed – (gTFR(t)+b_{20}Var)	1927-2009 (83)	.144	22	1	0
Observed – (gTFR(t)+b_{30}Var)	1932-2004 (73)	.145	18	1	0

NOTES: See discussion in text and Appendix Section G. The Observed value is always CFR(T). The Average Error is the sum of the magnitudes (absolute values) of the errors for each year divided by the number of years observed. The symbol b_{20} denotes the coefficient of the quadratic term in the cubic curve fitted to years −10, −5, +5, and +10 around each focal year, which was taken to be year 0. The symbol b_{30} denotes the coefficient of the quadratic term in the cubic curve fitted to years −15, −5, +5, and +15 around each focal year 0. The symbol gTFR represents the TFR series graduated by a 21-term moving average around each year.