Online Resource 2

Estimating Transition Rates

Because information about mortality rates of the nondisabled and the disabled was not available from primary data sources, we decomposed total mortality using the prevalence of disability and the hazard ratio of disability on mortality as follows:

$$\tilde{m}_{x,t}^{(nd,g)} = \frac{m_{x,t}^{(g)}}{\widehat{HR}_x^g \times \hat{p}_{x,t}^{(d,g)} + \left(1 - \hat{p}_{x,t}^{(d,g)}\right)} \tag{B1}$$

$$\tilde{m}_{x,t}^{(d,g)} = \tilde{m}_{x,t}^{(nd,g)} \times \widehat{HR}_x^g, \tag{B2}$$

where $m_{x,t}^g$, $\tilde{m}_{x,t}^{(nd,g)}$, $\tilde{m}_{x,t}^{(d,g)}$, $\widehat{HR}_x^g$, and $\hat{p}_{x,t}^{(d,g)}$ indicate the gender-specific (*g*) population mortality rate, the estimated mortality rates of the nondisabled and the disabled, the estimated hazard ratio, and smoothed prevalence of disability at age *x* and time *t*, respectively.

The converted transition, $q_{x,t}^{(nd,g)}$ ($q_{x,t}^{(d,g)}$), shows the probability that a person is nondisabled (disabled) at age *x* at time *t* and is dead at age *x* + 1 at time *t*. Such a formulation of the model implies the period-age approach, which is often used when the main point of interest is the change of transition probabilities over a certain period of time (e.g., calendar years). With the period-age approach, it is implicitly assumed that a person aged *x* at time *t* will have the same transition probability at age *x* + 1 and time *t* + 1 (assumption) as a person who is aged *x* + 1 at time *t* (reality). Making such assumption is unavoidable in estimating period life expectancies.

Estimating Incidence Rates

Given the prevalence of disability $p_{x,t}^{(d,g)}$, $p_{x+1,t}^{(d,g)}$ at age *x* and *x* + 1 and at time *t*, the mortality rates of the nondisabled $m^{(nd,g)}_{x,t}$ and the disabled $m_{x,t}^{(d,t)}$ aged *x* at time *t*, it is possible to calculate the corresponding incidence rate, $m_{x,t}^{(inc,g)}$, because these quantities are interrelated and mutually define each other. The prevalence of disability at age $x + 1$ and time t is expressed as the ratio of those who are disabled to those who are alive. However, this fraction is dependent on the number of transitions during age x. That is, the prevalence at age $x + 1$ is a function of the number of people alive ($l_{x,t}^{g}$), the prevalence of disability, and the transition probabilities at age x and time t:

$$p_{x+1,t}^{(d,g)} = \frac{l_{x,t}^{g} p_{x,t}^{(d,g)} - l_{x,t}^{g} p_{x,t}^{(d,g)} q_{x,t}^{(d,g)} + l_{x,t}^{g}(1 - p_{x,t}^{(d,g)}) q_{x,t}^{(inc,g)}}{l_{x,t}^{g} - l_{x,t}^{g}(1 - p_{x,t}^{(d,g)}) q_{x,t}^{(nd,g)} - l_{x,t}^{g} p_{x,t}^{(d,g)} q_{x,t}^{(d,g)}}. \tag{B3}$$

Expressing transition probabilities as functions of the transition rates, the incidence rate can be obtained by the following formula after rearranging (B3):

$$A^{g} = \frac{\hat{p}_{x+1,t}^{(d,g)}}{1 - \hat{p}_{x,t}^{(d,g)}} - \frac{\hat{p}_{x+1,t}^{(d,g)} \hat{p}_{x,t}^{(d,g)} \tilde{q}_{x,t}^{(d,g)}}{1 - \hat{p}_{x,t}^{(d,g)}} - \frac{\hat{p}_{x,t}^{(d,g)}}{1 - \hat{p}_{x,t}^{(d,g)}} + \frac{\hat{p}_{x,t}^{(d,g)} \tilde{q}_{x,t}^{(d,g)}}{1 - \hat{p}_{x,t}^{(d,g)}} \tag{B4}$$

$$\hat{m}_{x,t}^{(inc,g)} = \frac{A^{g}\left(1 + \frac{\tilde{m}_{x,t}^{(nd,g)}}{2}\right)\left(1 + \frac{\tilde{m}_{x,t}^{(d,g)}}{2}\right) - \hat{p}_{x+1,t}^{(d,g)} \tilde{m}_{x,t}^{(mrn,g)}\left(1 + \frac{\tilde{m}_{x,t}^{(d,g)}}{2}\right)}{1 + p_{x+1,t}^{(d,g)}\left(1 + \frac{\tilde{m}_{x,t}^{(d,g)}}{2}\right) - \hat{p}_{x+1,t}^{(d,g)} - \frac{A^{g}}{2}\left(1 + \frac{\tilde{m}_{x,t}^{(d,g)}}{2}\right)}. \tag{B5}$$

Then $q_{x,t}^{(inc,g)}$ shows the probability that a person is nondisabled at age x and time t and disabled at age $x + 1$ and time t. Although our model assumes that only incidence is possible, evidence suggests that people can recover from disability even at higher ages. Therefore, the probability of incidence in our model can be interpreted as a modified net incidence probability, which corresponds to the number of transitions from a nondisabled to a disabled state minus the number of transitions from a disabled to a nondisabled state, relative to the number of nondisabled people.