Online Resource 1

The original (mortality or actuarial) life table is a transition model in which observed death rates, within age interval, are the basis of probabilities of dying, and in which the main parameter of interest is the expectation of life. A multistate life table (MSLT) model is an extension of the original life table method. In an MSLT, in addition to “alive” and the absorbing “dead” states, at least one additional state is distinguished, typically between “perfectly healthy” and “dead”—for example, “disabled.” In contrast to a mortality table, a multistate life table model shows, for each age, not simply the probability that a person of that age will die before his or her next birthday but also the probability that a person of that age will move from one state to another. Correspondingly, an MSLT shows the remaining life expectancy and the proportion of the original birth cohorts still alive at different ages, but also the remaining life expectancy and proportion of people still alive in a given state. Furthermore, the population-average life expectancy can be decomposed into a weighted average of health expectancies, indicating the number of years people are expected to live in each health state.

Conversion of Rates Into Probabilities Taking Into Account Competing Risks

In a multistate life table model, the possible transitions are expressed by the matrix,

$$M_{x,t}^g \sum_{i=1}^{n} (X_i - \bar{X})^2 , \ M_{x,t}^g$$

standing for the transition rates between the several states, and where \(g\) denotes the gender. The matrix \(M_{x,t}^g\) refers to the chance of moving from the \(i\)th state to the \(j\)th state in infinitesimal time. However, instead of infinitesimal time intervals, one typically works with longer periods, like one year. In such a case, one refers to the transition probability matrix \(Q_{x,t}^g\), whose \(q_{ij}\) element indicate the transition probability that a person at age \(x\) at time \(t\) is in the \(i\)th state and in the \(j\)th state one year later. Assuming that the exposure is linear in age, one can convert the transition rates into the appropriate transition probabilities.
Transition Rates

\( \mathbf{M}^g_{x,t} \) is the matrix of transition rates at age \( x \) and time \( t \)

\[
\mathbf{M}^g_{x,t} = \begin{bmatrix}
    m^{(\text{inc},g)}_{x,t} + m^{(\text{nd},g)}_{x,t} & -m^{(\text{rec},g)}_{x,t} \\
    -m^{(\text{inc},g)}_{x,t} & m^{(\text{inc},g)}_{x,t} + m^{(\text{d},g)}_{x,t}
\end{bmatrix}
\]

where \( \text{inc}, \text{rec}, \text{nd} \) and \( d \) represents the transition from non-disabled to disabled, from disabled to non-disabled, from non-disabled to dead, and from disabled to dead, respectively.

Transition Probabilities

Transition probabilities are calculated using linear approximation: that is, all transitions (incidence, deaths) occur in the middle of the interval.

\[
\mathbf{Q}^g_{x,t} = \frac{I - \frac{1}{2} \mathbf{M}^g_{x,t}}{I + \frac{1}{2} \mathbf{M}^g_{x,t}} = \begin{bmatrix}
    1 - \left( q^{(\text{inc},g)}_{x,t} + q^{(\text{nd},g)}_{x,t} \right) & q^{(\text{inc},g)}_{x,t} \\
    q^{(\text{inc},g)}_{x,t} & 1 - \left( q^{(\text{rec},g)}_{x,t} + q^{(\text{d},g)}_{x,t} \right)
\end{bmatrix}
\]

\( \mathbf{Q}^g_{x,t} \) is the transition-probability matrix, consisting of elements \( q_{ij}(x,t) \), which represents the one-year probability that an individual with gender \( g \) alive at age \( x \) and time \( t \) will be in state \( j \) at age \( x + 1 \), and \( I \) is a \( 2 \times 2 \) identity matrix.

\[
q^{(\text{inc},g)}_{x,t} = \frac{m^{(\text{inc},g)}_{x,t}}{1 + \frac{m^{(\text{inc},g)}_{x,t} + m^{(\text{d},g)}_{x,t}}{2}}
\]

\[
q^{(\text{nd},g)}_{x,t} = \frac{m^{(\text{inc},g)}_{x,t} + m^{(\text{nd},g)}_{x,t}}{1 + \frac{m^{(\text{inc},g)}_{x,t} + m^{(\text{d},g)}_{x,t}}{2}}
\]

\[
q^{(\text{d},g)}_{x,t} = \frac{m^{(\text{d},g)}_{x,t}}{1 + \frac{m^{(\text{d},g)}_{x,t}}{2}}
\]
Note that in our model, recovery is set to zero, and incidence is considered as net incidence (real incidence minus real recovery).

Calculating the Number of Nondisabled and Disabled Persons Alive

*Number of Persons Alive*

\( l^g_{x,t} \) is the sum of the number of nondisabled (\( l^\text{nd},g_{x,t} \)) and disabled (\( l^\text{d},g_{x,t} \)) individuals alive at age \( x \) and at time \( t \):

\[
 l^g_{x,t} = \begin{bmatrix}
 l^\text{nd},g_{x,t} \\
 l^\text{d},g_{x,t}
\end{bmatrix} .
\]  \hfill (A6)

\( l^g_{x+1,t+1} \) is the sum the number of aged \( x + 1 \) individuals with gender \( g \) alive at time \( t + 1 \), expressed as a function of the number of individuals alive and transition probabilities at age \( x \) and time \( t \) (\( l^\text{nd},g_{x,t} \), \( l^\text{d},g_{x,t} \), \( q^{(p,g)}_{x,t} \)).

\[
 l^g_{x+1,t+1} = \begin{bmatrix}
 l^\text{nd},g_{x+1,t+1} \\
 l^\text{d},g_{x+1,t+1}
\end{bmatrix} = \begin{bmatrix}
 l^\text{nd},g_{x,t} - l^\text{d},g_{x,t} q^{(p,d)}_{x,t} & -l^\text{nd},g_{x,t} q^{(p,\text{nd})}_{x,t} \\
 q^{(p,d)}_{x,t} & q^{(p,\text{nd})}_{x,t}
\end{bmatrix} . \hfill (A7)

*Prevalence of Disability*

\( p^g_{x,t} \) is the prevalence matrix consisting of elements \( p^\text{nd},g_{x,t} \) and \( p^\text{d},g_{x,t} \), which represent the proportion of the gender-specific (\( g \)) population without disability (\( \text{nd} \)) and the proportion with disability (\( d \)) at age \( x \) and time \( t \).

\[
 p^g_{x,t} = \begin{bmatrix}
 p^\text{nd},g_{x,t} \\
 p^\text{d},g_{x,t}
\end{bmatrix} = \begin{bmatrix}
 l^\text{nd},g_{x,t} \\
 l^\text{d},g_{x,t}
\end{bmatrix} . \hfill (A8)

\( p^g_{x+1,t+1} \) is the gender-specific (\( g \)) prevalence matrix, expressed as a function of the number of individuals alive and transition probabilities at age \( x \) and time \( t \) (\( l^\text{nd},g_{x,t} \), \( l^\text{d},g_{x,t} \), \( q^{(p,g)}_{x,t} \)).
Calculating Life Expectancy of the Nondisabled and Disabled

$L_{x,t}^g$ denotes the gender-specific ($g$) average number of nondisabled and disabled individuals aged $x$ and alive at time $t$:

$$L_{x,t}^g = \left\{ \begin{array}{ll}
\frac{\hat{p}_{x+1,t}^{n(d,g)} + \hat{p}_{x+1,t}^{n(d,g)}}{2} & \text{if } x < \omega, \\
\frac{\hat{p}_{x,t}^{n(d,g)} + \hat{p}_{x,t}^{d,g}}{2} & \text{if } x = \omega,
\end{array} \right. \quad (A10)$$

where $\omega$ is the maximum attainable age.

$T_{x,t}^g$ denotes the cumulative average number of nondisabled and disabled individuals aged $x$ who were alive at time $t$:

$$T_{x,t}^g = \left[ \begin{array}{c}
\sum_{s=0}^{x} \frac{\hat{p}_{x+s,t}^{n(d,g)} + \hat{p}_{x+s,t}^{n(d,g)}}{2} \\
\sum_{s=0}^{x} \frac{\hat{p}_{x+s,t}^{d,g} + \hat{p}_{x+s,t}^{d,g}}{2}
\end{array} \right]. \quad (A11)$$

Gender-specific ($g$) life expectancy of the nondisabled and disabled are calculated as

$$e_{x,t}^g = \left[ \begin{array}{c}
e_{x,t}^{n(d,g)} \\
e_{x,t}^{d,g}
\end{array} \right]. \quad (A12)$$
Disability-free life expectancy (DFLE) and life expectancy with disability (LwD) are calculated as

$$\begin{bmatrix} DFLE_{x,j}^{(g)} \\ LwD_{x,j}^{(g)} \end{bmatrix} = \begin{bmatrix} \frac{T_s^{(d,g)}}{\hat{r}_{x,j}^{(g)}} \\ \frac{T_s^{(d,g)}}{\hat{r}_{x,j}^{(g)}} \\ \frac{T_s^{(d,g)}}{\hat{r}_{x,j}^{(g)}} \end{bmatrix}.$$  \hspace{1cm} (A13).